



Reg. No. :

Name :

**Third Semester B.Tech. Degree Examination, April 2015
(2013 Scheme)**

**13.301 : ENGINEERING MATHEMATICS – II
(ABCEFHMNPRSTU)**

Time : 3 Hours

Max. Marks : 100

- Instructions :** 1) Answer **all** questions from Part A. Each question carries 4 marks.
2) Answer **one full** question from each Module of Part B. Each full question carries 20 marks.

PART – A

- Find the angle between the normals to the surface $xy = z^2$ at the points $(-2, -2, 2)$ and $(1, 9, -3)$.
- Find the workdone by the force $\vec{F} = x\hat{i} + 2y\hat{j}$ in moving a particle from $(0, 0)$ to $(2, 2)$ along the curve $2y = x^2$.
- Find the Fourier cosine transform of e^{-5x} .
- Form the partial differential equation by eliminating the arbitrary functions from

$$z = f(y + 3x) + g(y - 3x) + \frac{x^3 y}{6}$$

- Solve the equation $U(x, t) = e^{-t} \cos x$ with $U(x, 0) = 0$ and $\frac{\partial u}{\partial t}(0, t) = 0$ by the method of separation of variables.

PART – B

Module – I

- Show that $\vec{F} = (z^2 + 2x + 3y)\hat{i} + (3x + 2y + z)\hat{j} + (y + 2xz)\hat{k}$ is irrotational but not solenoidal.
 - If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ show that $\nabla^2 r^n = n(n+1)r^{n-2}$.
 - Use Divergence theorem to evaluate $\iint_S \vec{F} \cdot \hat{n} \, ds$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.
- Find the constants a, b, c so that $\vec{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.
 - If $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$. Find $\int_C \vec{F} \cdot d\vec{r}$ where C is the rectangle in the xy plane bounded by $x=0, x=a, y=0, y=b$.
 - Use Green's theorem in the plane to evaluate $\int_C (x^2 - 2xy) \, dx + (x^2 y + 3) \, dy$ where C is the boundary of the region bounded by $y = x^2$ and $y = x$.





Module – II

8. a) Find the Fourier series of period $2l$ for the function

$$f(x) = l - x, 0 \leq x \leq l \\ = 0, \quad l \leq x \leq 2l$$

- b) Expand $f(x) = \pi x - x^2$ as a half range sine series in the range $(0, \pi)$.
- c) Find the Fourier transform of $f(x) = x, |x| < a$
 $= 0, |x| > a, a > 0.$
9. a) Find the Fourier series of $f(x) = x - 2x^2, -\pi < x < \pi.$
- b) Expand $f(x) = x + \pi, 0 \leq x \leq \pi$
 $= -x + \pi, -\pi \leq x \leq 0,$ given that $f(x)$ is periodic with period $2\pi.$
- c) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$

Module – III

10. a) Solve $\frac{\partial^2 z}{\partial x^2} + a^2 z = 0,$ given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = a.$
- b) Find the singular integral of $z = px + qy + p^2 - q^2.$
- c) Solve : $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2).$
11. a) Solve the equation $(D^4 - D'^4)z = e^{x+y}.$
- b) Solve : $z = xp^2 + qy.$
- c) Solve : $yp = 2xy + \log q.$

Module – IV

12. a) Find the variable separable solution of the heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}.$
- b) If a string of length ' l ' is initially at rest in the equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = V_0 \sin^3 \frac{\pi x}{l}, 0 < x < l.$ Find the displacement $y(x, t).$
13. a) A tightly stretched string of length ' l ' is fastened at both ends. Motion is started by displacing the string into the form $kx(l - x)$ from which it is released at time $t = 0.$ Find the displacement $y(x, t).$
- b) A rod of length ' l ' has its ends A and B kept at 0°C and 80°C respectively until steady-state conditions prevail. If B is suddenly reduced to 0°C and kept so, while that of A is maintained, find the temperature function $u(x, t).$